The Rational Method

Introduction

The Rational Method at age 150 remains yet the most-employed formula of engineering hydrology. The Rational Method is logical, generalized and often reasonable. Yet its theoretical basis is significantly violated in most of its applications. This course explores the practical realm of hydrologic analysis, bounded by multitudes of complex natural interactions on one hand and the stark non-negotiable simplicity of physical law on the other. The Rational Method is both art and science.

The Rational Method is presented from the perspective of each of the Rational Method's three “independent” variables. You will quickly discern that these variables intertwine. To understand one, you must understand the others.

History

More than a tale about a formula, the genesis of the Rational Method is part of the larger ascent of the scientific perspective.

Not until Pierre Parrault (a French attorney prone to embezzlement) published “De l’origine des fontaines” in 1674 was it quantitatively established that stream flow derives from precipitation in the contributing watershed. Parrault’s observation rectified 2000 years of Greek-inspired philosophical confusion regarding a hydrologic cycle in which rivers are ocean-fed from subterranean seas or earth-respired vapors. Parrault showed hydrology to be a quantifiable, physics-based discipline.

It took another 170 years for investigators, primarily in the British Isles, to conclude that the ratio of runoff to rainfall, at least for a particular watershed, might be approximated as a coefficient, typically in the 0.4-0.6 range for natural catchments. Such a coefficient is the first principle of what would become known as the Rational Method.

In 1851, Thomas Mulvaney presented the Rational Method’s second principle, the role of the runoff’s time of concentration in quantifying the storm event, to the Irish Institution of Civil Engineers. Mulvaney proposed that,

$$Q = C i_{ave} A_{cont} \quad \text{Eq.1}$$

where Q (in Imperial units) is watershed runoff rate in cubic feet per second (cfs), C is a dimensionless runoff-to-rainfall coefficient between 0.00 and 1.00, i-ave is rainfall intensity in inches/hour averaged over the time of concentration and A-cont is contributing watershed area in acres. (Because the verbiage of this course may find itself reduced to text-only formatting, subscripts will be designated with a dash.) Further definition of the three right-hand-side variables is the bulk of this course.

Mulvaney’s clock-driven recording rain gage, an instrument required to calibrate the Rational Method, moved the development from theory to practice. The method is also known as the Kuichling formula in the United States in honor of Emil Kuichling who applied it for sewer design in Rochester, NY, 1877-1888. The method is known as the Lloyd-Davies formula in the United Kingdom in honor of D.E. Lloyd-Davies who wrote about it in 1906.

The obviousness of the hydrologic cycle today attests to the obliviousness to measurement only several-hundred years ago. The Rational Method was one step in the transformation. Unfortunately, as the body of this course details, because that step could mathematically achieve reasonable results (given the often-poor limits of verification), that step has too often been the only one taken. Most engineering techniques are revised, if not revolutionized, every few generations. Mulvaney's work persists.

The Rational Method has been heavily employed for a century in much of the developed world’s urban construction. The method's basis, however, has no more verification than that of one small parking lot study by Johns Hopkins University (ASCE, 1996, p. 582). Mulvaney's work persists because of its ease, not its confirmation.

Why the Method is "Rational"

The Rational Method is misdesignated "rational" by those who attribute the title to fundamental physical reason. As will be discussed, the method's logic is highly idealized. FHWA (1986, p. 137) classifies the method as empirical. The rational mind should at least suspect that Eq. 1 is analytically superficial. To its "rational" credit, the method, in its infancy, was less arbitrary than its competitors.

Eq. 1 is unsystematically dimensional, employing feet, seconds, inches, hours and acres. If one takes each right-hand-side variable as 1.0, runoff is 1.0 acre-inch/hour, which converts to 1.00833 cfs, 1 cfs in any practical sense. Some authors thus attribute the title "rational" to the closeness of 1.00833 to unity. If that were the basis, the equation might more appropriately be known as the "Coincidental Imperial Units Method", as Eq. 1 can, of course, be formulated for any dimensional system by an appropriate constant. In SI form, Eq. 1 becomes,

$$Q = 0.00278 C i_{ave} A_{cont} \quad \text{Eq. 2}$$

where Q is in cubic meters per second, i-ave is in millimeters/hour and A-cont is in hectares. The 0.00278 obviously isn't close to unity. This course will use Imperial units, Eq. 1.

The Rational Method is "rational" by virtue of its reliance on a ratio C of runoff to rainfall intensity. The method title would be less ambiguous if "nal" were deleted.

The United States Army knows the wisdom of strategic retreat. USACE, (1996, p. 110) ducks the etymologic (word history) question entirely, calling the computation "the so-called rational method". If the professional community cannot agree on why something is named, it should come as no surprise when the community doesn't agree on its employment.

C as a Scientific Parameter

As will be immediately evident, a C is by no means a constant in the manner of pi, an immutable value. C is a variable. The dilemma manifests itself in most applications of the Rational Method. A reference may indicate that C for, say, heavy industrial zones lies between 0.60 and 0.90 or perhaps is typically 0.80. What contributes to the variability? Can such contributions be separated?

In a mechanical sense, C is the product of two coefficients, Eq. 3,

$$C = C_a C_d \quad \text{Eq. 3}$$

C-a is due to abstractions (infiltration, interception, detention storage, etc.) and C-d is related to diffusion, the tendency for a hydrograph to spread in time and attenuate in peak as it moves downstream. Hydraulic engineers see C-d as a "routing" issue. Both C-a and C-d are dimensionless. Neither can exceed unity. From the practical viewpoint, as we will see in Section 9, C-d is treated as if it were 1.00 and C-a can be seen as,

$$C_a = \frac{i - \phi}{i} \quad \text{Eq. 4}$$

where i-ave is rainfall intensity measured in inches/hour and phi is an index of loss rate in similar units. The hydrologic "phi index" can be used to horizontally divide a hyetograph into direct runoff above and abstraction below. If no loss occurs, phi is 0.0 and C is 1.00. If loss is at the same rate as rainfall, C is 0.00. Loss rate can't exceed rainfall rate because of mass conservation. From the perspective of science, however, Eq. 4 leads nowhere because phi is a one-event empirical backcalculated watershed parameter.

Alternatively, where a watershed's flow path is complex, the C can be seen as an expression of two empirical components, one C relating to overland flow and another C relating to channel flow Singh (1988, p. 127) provides some guidance, but not enough for scientific application.

Chadwick and Morfett (1993, p. 328) propose that C is the product of a volumetric runoff coefficient C-v and a routing coefficient C-r. But as the latter is an unexplained constant 1.3 and the former is a regressed “urban percentage runoff”/100, true science again seems to be illusionary.

What’s above is indicative of C's bleak scientific underpinning. C stems from some combination of physical processes, to be sure, but we’ve little explanation for such physics.

<u>C as an Empirical Parameter</u>

What follows indicates empirical reality, bewildering perhaps in small doses, but rich when viewed in total. C is best quantified by the art of watershed observation.

Tables summarize C for various land treatments, the latter term not necessarily implying human intervention. Many of the slope-unspecified values are reprinted from one reference to the next for more than 50 years. Check your hydrologic references for “Railroad Yards” in the C table. If you find it, you have this data set. Railroads aren't as big of concern today.

The topic of return periods is addressed later in this chapter, but for reference, Tables 1 and 2 represent 2 to 10-year T-r’s according to ASCE (1996, p. 591) or 5 to10 years according to ODOT (1990, p. 11). The values simply represent typical values for storms of unremarkable magnitude.

Table 1. Rational C's, Urban Conditions					
			Slope		
			< 0.02	0.02-0.10	> 0.10
Industrial	light	0.50-0.80	0.50	0.70	0.80
	heavy	0.60-0.90	0.60	0.80	0.90
Business	downtown	0.70-0.95	0.80	0.85	0.85
	neighborhood	0.50-0.70			
Residential	single-family	0.30-0.50			
	multi-units, detached	0.40-0.60			
	multi-units, attached	0.60-0.75			
	suburban	0.25-0.40			
	1-3 units/ac		0.35	0.40	0.45
	3-6 units/ac		0.50	0.55	0.60
	6-15 units/ac		0.70	0.75	0.80
	apartments	0.50-0.70	0.50	0.60	0.70
Park & Cemetery		0.10-0.25	0.10	0.15	0.25
Playground		0.20-0.35	0.20	0.25	0.30
Drive & Walk		0.75-0.85	0.75	0.80	0.85
Railroad Yard		0.20-0.35			
Roofs		0.75-0.95			
Water Impoundment		1.00			
Lawn	soil unspecified		0.17	0.22	0.35
	sandy soil		0.05-0.10*	0.10-0.15*	0.15-0.20*
	heavy soil		0.13-0.17*	0.18-0.22*	0.25-0.35*
Roadway	unspecified asphalt	0.70-0.95	0.90	0.90	0.90
	concrete	0.70-0.95			
	brick	0.70-0.85			
	gravel		0.50	0.55	0.60
	earth shoulder		0.50	0.50	0.50
	grass shoulder		0.25	0.25	0.25
	earth sideslope		0.60	0.60	0.60
	turf sideslope		0.30	0.30	0.30
	turf median		0.25	0.30	0.30
*Slope < 0.02, 0.02-0.07, > 0.07					
Sources: Chow, 1964, p.14-7, 21-38; ASCE, 1996, p. 591; ODOT, 1990, p. 11					

Table . Rational C's, Rural Conditions					
			Slope		
			< 0.05	0.05-0.10	> 0.10
Cultivated	sand & gravel		0.25*	0.30*	0.35*
	sandy loam	0.20	0.30	0.40	0.52
	clay & loam		0.50*	0.55*	0.60*
	clay & silt loam	0.40	0.50	0.60	0.72
	tight clay	0.50	0.60	0.70	0.82
Pasture, Range, Meadow			0.25*	0.30*	0.35*
	sandy loam	0.15	0.10	0.16	0.22
	clay & silt loam	0.35	0.30	0.36	0.42
	tight clay	0.45	0.40	0.55	0.60
Woodland, Forest			0.10*	0.15*	0.20*
		0.30	0.10	0.20	0.30
	sandy loam	0.10	0.10	0.25	0.30
	clay & silt loam	0.30	0.30	0.35	0.50
	tight clay	0.40	0.40	0.50	0.60
Slope < 0.02, 0.02-0.10, > 0.10					
Sources: Chow, 1964, p. 21-38; ASCE, 1996, p. 590)					

For bounded ranges, McCuen (1998, p. 377) recommends intermediate values with 0.05 precision, either halfway between the bounds or rounded up.

Fig. 1 shows C as a function of residential dwelling units per acre (King Co., 1998, p. 3-13). Also shown is the decimal impervious Imp including local streets (City of Albuquerque, 1997, p. 22-11). As might be anticipated, C and Imp are closely related.

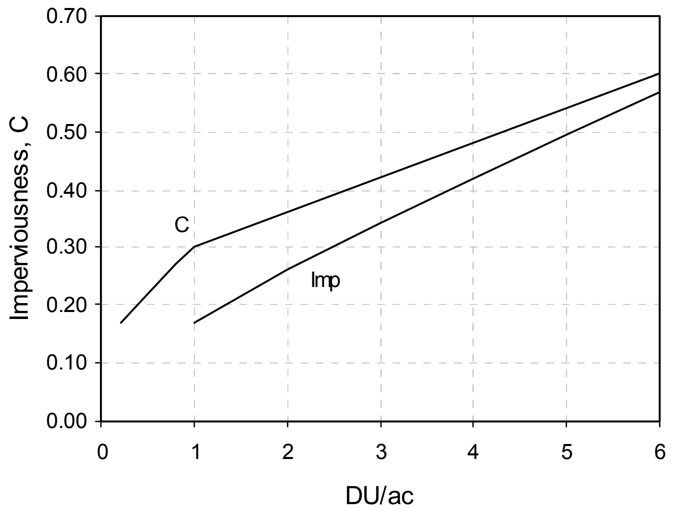

Figure 1. Single Family Density, Imp and C

FIG 1. DU's

Fig. 1.1 shows the tie between C and Imp from experimental comparison. Maidment (1993 p. 28.2) reports a third-order polynomial fit, but the general trend and the individual spread together suggest a relationship that in an approximate sense is linear.

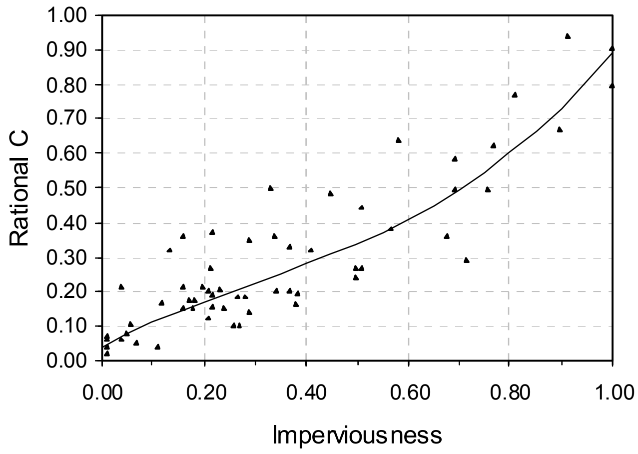

Figure 1.1. Imperviousness and Rational C

FIG. 1.1 C AND IMP

As FHWA (1996, p. 3-6) notes, "Higher values are usually appropriate for steeply sloped areas…. because infiltration and other losses have a proportional smaller effect on runoff in these cases." As can be seen, C increases with slope. Faster runoff has less opportunity to disappear.

According to Chow (1964, p. 21-38), "Watershed slope is not directly considered in the selection of C, but the effects of slope are included, since rainfall duration is taken as equal to the time of concentration." While Chow's attempt to separate slope from C has its theoretical merit, most data sets, ours included, don't follow that guidance.

The influence of slope appears again in Section 5 where it impacts time of concentration. While the reasoning there relates to i-ave, not C, the impact on Q is the same. Q goes up.

<u>C Components</u>

Tables 3 and 4 partition C into components. One could presumably reduce much of Figs. 1-6 to similar rule bases, but it hasn't been done. Table 3 partitions C into three factors. C is 1.00 minus their sum. For flat, medium clay and loam cultivated land, C is 1.00 – 0.30 – 0.20 – 0.10 = 0.40.

Table 3. Deductions from Unity to Obtain Rational C, Agricultural Areas	
Factor	C subtraction
Topography	
Flat, average S = 0.0002-0.006	0.30
Rolling, average S = 0.003-0.004	0.20
Hilly, average S = 0.03-0.05	0.10
Soil	
Tight impervious clay	0.10
Medium combinations of clay and loam	0.20
Open sandy loam	0.40
Cover	
Cultivated	0.10
Woodland	0.20
Source: Gray (1970), p. 8.5	

Table 4 provides another tabular estimate. These components are summed. C for grassland with 0.02 slope, a 100-year T-r and 50 inch mean annual precipitation is 0.35 – 0.05 + 0.05 + 0.03 = 0.38.

Table 4. Rational C, Rural Areas	
Factor	C partial
Ground Cover	
Bare surface	0.40
Grassland	0.35
Cultivated	0.30
Timber	0.18
Slope	
< 0.05	- 0.05
> 0.10	+ 0.05
Return Period	
< 20 years	- 0.05
> 50 years	+ 0.05
Mean Annual Precipitation	
< 24 inches	- 0.03
> 36 inches	+ 0.03
Source: Gupta (1989), p. 621	

Component approaches such as those of Tables 1 and 2 make clear the relative importance of factory. The approach does not catch the significance of interactions, however.

<u>C and NRCS Considerations</u>

Table 5 introduces Hydrologic Soil Group (HSG), a Natural Resources Conservation Service (formerly the Soil Conservation Service) soil classification. HSG A is relatively permeable. HSG D is relatively impermeable. HSG B and C fall between. NRCS has accordingly mapped much of the United States. Chapter 6 contains more information. C increases as HSG progresses from A to D, the logical consequence of decreasing infiltration capacity. The last “Pervious” land treatment is for Erie and Niagara Counties (Debo and Reese, 1995, p. 217). The others are less site-specific. Table 5 suggest that HSG’s influence upon C is predictably consistent.

Table 5. Rational C's, T-r < 25 yr, >= 25 yr						
	Slope					
	< 0.02		0.02-0.06		> 0.06	
HSG	A C	B D	A C	B D	A C	B D
1/8-acre Lots	25, 33 30, 38	27, 35 33, 41	28, 37 33, 42	30, 39 36, 45	31, 40 38, 49	35, 44 42, 54
1/4-acre Lots	22, 30 27, 36	24, 33 30, 38	26, 34 31, 40	29, 37 34, 42	29, 37 36, 47	33, 42 40, 52
1/3-acre Lots	19, 28 25, 33	22, 30 28, 36	23, 32 29, 38	26, 35 32, 40	26, 35 34, 45	30, 39 39, 50
1/2-acre Lots	16, 25 22, 31	19, 28 26, 34	20, 29 27, 35	23, 32 30, 38	24, 32 32, 42	28, 36 37, 48
1-acre Lots	14, 22 20, 28	17, 24 24, 31	19, 26 23, 32	21, 28 29, 35	22, 29 31, 40	26, 34 35, 46
Industrial	67, 95 68, 86	68, 85 69, 86	68, 85 69, 86	68, 86 69, 86	68, 86 69, 87	69, 86 70, 88
Commercial	71, 88 72, 89	71, 89 75, 89	71, 88 72, 89	72, 89 72, 89	72, 89 72, 90	72, 89 72, 90
Streets	70, 76 72, 84	71, 80 73, 89	71, 77 73, 85	72, 82 75, 91	72, 79 76, 89	74, 84 78, 95
Parking	85, 95 85, 95	85, 95 85, 95	86, 96 86, 96	86, 96 86, 96	87, 97 87, 97	87, 97 87, 97
Cultivated Land	8, 14 14, 20	11, 16 18, 24	13, 18 19, 25	15, 21 23, 29	16, 22 26, 34	21, 28 31, 41
Pasture	12, 15 24, 30	18, 23 30, 37	20, 25 34, 42	28, 34 40, 50	30, 37 44, 52	37, 45 50, 62
Meadow	10, 14 20, 26	14, 20 24, 30	16, 22 28, 35	22, 28 30, 40	25, 30 36, 44	30, 37 40, 50
Forest	5, 8 10, 12	8, 10 12, 15	8, 11 13, 16	11, 14 16, 20	11, 14 16, 20	14, 18 20, 25
Open Space	5, 11 12, 18	8, 14 16, 22	10, 16 17, 23	13, 19 21, 27	14, 20 24, 32	19, 26 28, 39
Pervious.	4, 9 11, 16	7, 12 15, 20	9, 14 16, 21	12, 17 20, 25	13, 18 23, 31	18, 24 28, 38
Source: McCuen, 1998, p. 376						

To keep the reference data together, Table 5 also shows return period T-r up to 25 years and 25 years or more. T-r is analytically pursued in Section 6, so for now may simply be taken as presented. C increases, again with reasonable predictability, with T-r.

NRCS Curve Number CN estimates runoff volume, the subject of Chapter 6. The Rational Method estimates runoff rate. CN and C are not easily related, but not for lack of effort. C for an infinite sponge is 0.00. CN is 0. C for a sheet of glass is 1.00 if wave diffusion is ignored. CN is 100. Such convenient agreement between end points suggests an erroneous linear correspondence between. CN 50 often produces no runoff while a 0.50 C produces runoff at half the precipitation rate.

The Rossmiller equation provides one C estimate based on CN (Viessman and Lewis, 1999, p. 316). Eqs. 5 and 6 are formulated to emphasize the parametric roles.

$$C = \frac{1.573 * 10^{-7} T_r^{0.05} (Imp + 1)^{0.7} CN^K}{\left(9.677 * 10^{-4}\right)^{S^{0.2}} 0.5012^i} \qquad \text{Eq. 5}$$

$$K = 3.222 - 1.5071 S^{0.2} - 0.148 i \qquad \text{Eq. 6}$$

where CN is the Curve Number, T-r is return period in years, Imp is decimal impervious, S is dimensionless slope and i-ave in rainfall intensity in inches/hour. The irrational constants derive from mathematical reductions. If T-r is 10 years, Imp is 0.2, CN is 87, S is 0.04 and i-ave is 1.0 inch/hour, K is 2.2823 and C is 0.41. Dropping CN to 77, C is 0.31. Eqs. 5 and 6 are appropriate only within narrow bounds. The safe prediction is that that high C's are likely to be found where CN's are also high. Don't venture beyond that unless the NRCS forces you to.

C varies with soil moisture, Antecedent Moisture Condition (AMC I = dry, II = normal, III = wet) being the NRCS specification. An AMC I may have a C of 0.05 and an AMC III may have a C of 0.60 for what otherwise appears to be identical storms. It is unfortunate that C's dependence on AMC has received little analysis. Chadwick and Morfett (1993, p. 328) present a U.K. alternative, in rearranged form,

$$C = 1.3\left[\frac{82.9Imp + 25.0SOIL + 0.078UCWI - 20.7}{100}\right] \quad \text{Eq. 7}$$

where Imp is decimal impervious (Table XX, Chapter 3), SOIL is a number depending on soil type and UCWI is the urban catchment wetness index (mm) related to mean annual rainfall. In Chadwick and Morfett's example, Imp is 0.35, SOIL is 0.3 and UCWI is 72 mm. C is thus 0.28. While SOIL and UCWI parameters would not be available for most applications, Eq. 7 does illustrate C's dependency on soil moisture.

C and i-ave

An often-noted dependency is that of C to i-ave, the latter being an inverse function of t-d. As C inversely varies with t-d, C should increase as i-ave increases. According to Gray (1970, p. 8.4), "Unfortunately, quantitative information of the influence of intensity on C is completely lacking." In fact, quantitative information is not completely lacking, as evidenced by site-specific Figs. 2 (Ponce, 1989, p. 123) and 3 (Singh, 1988, 123).

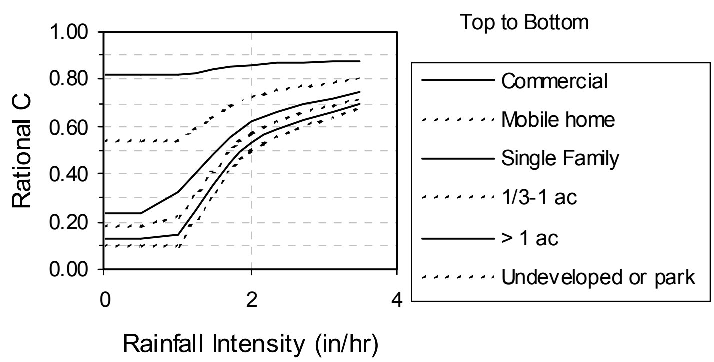

Figure 2. Rational C's and Rainfall Intensity

FIG. 2. INTENSITY

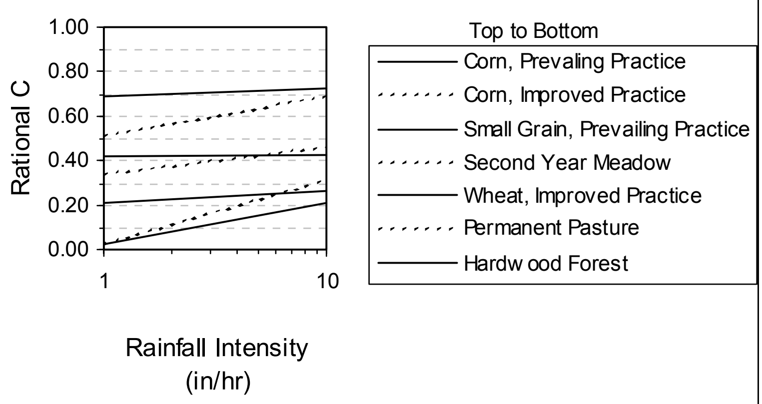

Figure 3. Rational C's and Rainfall Intensity, Choshocton, Ohio

FIG. 3. CHOSHOCTON

Figs. 2 and 3 slope upward as expected. A given depth of rainfall over a shorter t-d (thus of greater i-ave) has a greater C. As abstraction is generally limited by soil and/or interception capacity, this makes hydrologic sense. Eq. 8, the Roe and Snyder modification for small agricultural watersheds, makes the Rational Method exponential in i-ave.

$$Q = C i_{ave}^{a} A_{cont} \qquad \text{Eq. 8}$$

where a is 1.14 and C varies according to the watershed groups in Table 6.

Table 6. Roe and Snyder Coefficient C

Group	i-ave (in/hr)	t-d (min)	A (ac)	C
I	< 0.5	ave 114	75-373	0.60
II	> 1.5	ave 38	21-112	0.36
III	< 1.0	ave 51	75-373	0.22
IV	> 1.0	ave 33	75-373	0.07

Source: Singh, 1988, p. 140.

UDFCD (2001, p. 25) approximates C as the ratio of runoff depth Q to the 2-hour rainfall depth P-2 (Eq. 8.5) which in turn relates C to the respective I-ave (Eq. 8.6).

$$C = \frac{Q}{P_2} \qquad \text{Eq. 8.5}$$

$$C = \frac{Q}{2 i_{ave,2hr}} \qquad \text{Eq. 8.6}$$

While Eq. 8.6 looses the Rational Method’s crucial general-case i-ave meaning, it may provide a quick estimate.

If one were to match the Eq. 8 result by adjusting C and keeping exponent a at 1.00, C would appear to increase with i-ave. If, however, C has already been adjusted for t-d via another route, don't doubly correct.

<u>C and Timing</u>

C varies with length of time prior to thorough wetting of the soil. C modification begins at the start of t-c. Chow (1964, p. 20-8) mentions “some designers use values of C varying with length of time prior to a thorough wetting of the soil." Eq. 9 does this, relating C to degree of imperviousness and timing within the storm,

$$C = \frac{0.98t}{4.54 + t} C' + \frac{0.78t}{31.17 + t} (1 - C') \quad \text{Eq. 9}$$

where C' is the C uncorrected for antecedent rainfall and t is the time in minutes from beginning of rainfall to the beginning of t-d (Gupta, 1989, p. 620). Using Gupta's example, C' is 0.65 and the 20-minute t-d begins 80 minutes into the storm. Corrected C is 0.79 when t-d begins (t = 80) and 0.81 when t-d ends (t = 100). C for the event is the 0.80 average. Beware of this example, however, as the 0.65 is for a condition we would unlikely know. What Eq. 9 best shows is how the C creeps up during an event

C and Return Period

Most references append a T-r note to C tables. Chow (1964, p. 14-7) states that C is higher than those tabled for storm events exceeding a 5 or 10-year T-r, but does not speculate how much higher.

Table showed C for T-r's less than 25 years and larger C's for T-r's equalling or exceeding 25 years. The larger C was often about 25 percent higher, reasonably in accord with Table 7's multipliers.

Table 7. Rational C T-r Multipliers	
T-r (years)	Multiplier
2-10	1.0
25	1.1
50	1.2
100	1.25
Source: Viessman & Lewis (1996), p. 315	

When applied to a C, the Table 6 product cannot exceed 1.00. If a 5-year C is 0.90, the 50-year C can't be 1.2 * 0.90 = 1.08. It's 1.00.

Chow (1964, p. 20-9) and Singh (1988, p. 140) cite the Bernard modification, Eq. 10,

$$C = C_{100} \left(\frac{T_r}{100} \right)^x \quad \text{Eq. 10}$$

where T-r is return period in years, C-100 is C for the 100-year event and x is a geographical constant. If x is 0.075, Eq. 10 approximates Table 7. Chow calls this the "general rational formula" as if T-r alone pulls all things together.

Table 8 shows the impact of T-r on C for Austin, TX. This data confirms behaviors seen earlier related to slope, vegetation and development.

Table 8. Rational C and Return Period								
		T-r (yr)						
		2	5	10	25	50	100	500
Lawn and Impervious								
< 50% grass cover	< 0.02	0.31	0.34	0.37	0.41	0.44	0.48	0.57
	0.02-0.07	0.37	0.40	0.43	0.46	0.49	0.53	0.61
	> 0.07	0.40	0.43	0.45	0.49	0.52	0.55	0.62
50-75% grass cover	< 0.02	0.25	0.28	0.30	0.34	0.37	0.41	0.53
	0.02-0.07	0.33	0.36	0.38	0.42	0.45	0.49	0.58
	> 0.07	0.36	0.39	0.42	0.45	0.48	0.52	0.60
> 75% grass cover	< 0.02	0.21	0.23	0.25	0.29	0.32	0.36	0.49
	0.02-0.07	0.29	0.32	0.35	0.39	0.42	0.46	0.56
	> 0.07	0.34	0.37	0.40	0.44	0.47	0.51	0.59
Asphalt		0.73	0.77	0.81	0.86	0.90	0.95	1.00
Concrete, Roofs		0.75	0.80	0.83	0.88	0.92	0.97	1.00
Undeveloped Land								
Cultivated	< 0.02	0.31	0.34	0.36	0.40	0.43	0.47	0.56
	0.02-0.07	0.35	0.38	0.40	0.44	0.47	0.51	0.58
	> 0.07	0.39	0.42	0.44	0.48	0.51	0.54	0.61
Pasture, Range, Meadow	< 0.02	0.25	0.28	0.30	0.34	0.37	0.41	0.53
	0.02-0.07	0.33	0.36	0.38	0.42	0.45	0.49	0.57
	> 0.07	0.37	0.40	0.42	0.46	0.49	0.53	0.60
Woodland, Forest	< 0.02	0.22	0.25	0.28	0.31	0.35	0.39	0.48
	0.02-0.07	0.30	0.33	0.35	0.39	0.42	0.46	0.55
	> 0.07	0.36	0.39	0.41	0.45	0.48	0.52	0.59
Source: Chow, Maidment and Mays, 1988, p. 498								

Recall that Table 7 recommends increasing a 2 or 10-year C by 25 percent to estimate the 100-year value. Table 8 suggests something nearer 40 percent.

Fig. 15 shows an alternative T-r adjustment to C. In this method, C-max is effectively the same as C-100 in Eq. 10. Start with C for a known T-r, find C-max graphically and use that value to compute C for another T-r. A 90 percent impervious watershed has a C for a 10-year event (C-10) of 0.86. What’s C-50? C-10/C-max is 0.83. C-50/C-max is 0.98. C-50/C-10 is thus 1.18. This multiplied by 0.86 is 1.02. As this exceeds 1.00, C-50 is 1.00.

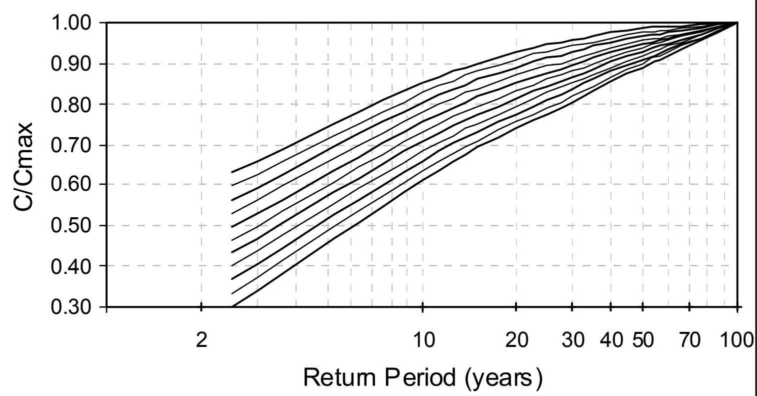

Figure 4. Return Period Correction (Gupta, 1989, p. 619)

FIG. 4. T-r CORRECTION

Maidment (1993, p. 19-8) presents a “probabilistic approach” to C vs. T-r, an empirical regional formulation employed in Australia. Fig. 16's two frequency plots illustrate the first part of the approach. The watershed would need to have been instrumented for both rainfall intensity and runoff rate over a long period. The intensity is i-ave for the known t-c. Backcalculated C’s increase from 0.58 to 0.76 for 2 to 500-year T-r’s, the trend anticipated in this section. The method is probabilistic in the sense that the data record conveys the stochasticity of nature. The second part of the approach pursues spatial distribution of C’s when multiple watersheds are mapped.

Figure 5. Intensity Return Periods

FIG. 5. INTENSITYT-r

Average rainfall intensity fro the Seattle area is estimated by Eq. 10.1.

$$i_{ave} = \frac{aP_{24}}{t_c^{\ b}} \qquad \text{Eq. 10.1}$$

where i-ave is in inches/hour, P-24 is the 24-hour storm depth in inches for the appropriate T-r, t-c is time of concentration in minutes between 6.3 and 100, and a and b are coefficients as given in Table 8.1

Table 8.1. Rational C a and b Coefficients		
T-r (years)	a	b
2	1.58	0.58
5	2.33	0.63
10	2.44	0.64
25	2.66	0.65
50	2.75	0.65
100	2.61	0.63
Source: King Co., 1998, p. 3-13		

A 100-year event of 3-inch depth has an 1.19-inch/hour i-ave for a 20 minute t-c.

Unaddressed C Factors: Area and Hydraulics

Yield (cfs/acre) is generally inversely related to watershed area. An illustrative 100-acre watershed yields 25 cfs for a protracted 0.5-inch/hour storm. C thus must be 0.50. The same storm produces 75 cfs for a 500-acre basin. Runoff finds more opportunity for abstraction along the longer route and the hydrograph has more time to diffuse. If C is 0.50, however, Eq. 1 predicts 125 cfs for 500 acres. To hit the 75 cfs, C for 500-acres must be 0.30. C would benefit from an area correction in the manner that point precipitation depths are reduced when spread over a large watershed (ASCE, 1996, p, 58). What is done in practice, however, is to dodge the area correction by confining the Rational Method to relatively small areas. We'll look at this again in Section 8.

A scientific C should, but doesn't, incorporate the hydraulics of backwater (Debo and Reese, 1995, p. 3) or temporary ponding (ASCE, 1992, p. 91 and Maidment, 1993, p. 9.15). The Rational Method is oblivious to the perspective of fluid mechanics or physically-based hydrologic simulation. Watershed aspects that complicate the hydraulics probably also complicate the C. Proceed with care.

<u>Composite C</u>

Many watersheds are not of a single land treatment. Eq. 11 is an area-weighted composite C for a watershed spatially distributed by subareas.

$$C_{comp} = \frac{\sum_{i=1}^{n} C_i A_i}{A_{tot}} \qquad \text{Eq. 11}$$

where the watershed has n subareas, C-i and A-i are C and A-cont of subarea i. A-tot is the sum of A-i's. We will employ Eq. 11 when we subdivide a watershed by isochrones later in this chapter.

If a 20-acre watershed has subareas of 5, 6, 6 and 3 acres with respective C's of 0.40, 0.30, 0.40 and 0.10, C-comp is 0.325. A football team that averages X pounds isn't the same as a team where each player weighs X. Likewise, runoff from a 0.325 uniform C watershed may not be the same as the combined runoff from the 4-subareas. Be aware of averaging's effect on both timing and abstracts.

<u>Average Intensity and Time of Concentration</u>

Chapter 1 employed a variety of graphics describing a rainfall event. The Rational Method utilizes but a small portion of that information, the average intensity over a specified duration. Chapter 1 illustrated this measure in several figures: 1.39 inches/hour for 30 minutes. The Rational Method sees this i-ave no differently than that of a storm that rains 13.9 inches over 10 hours at a uniform rate. We'll return to the issue, but you should already be questioning if the two storms, each with a 1.39 inches/hour i-ave, really have the same Q.

For many Rational Method calculations, we can stick with the water travel t-c viewpoint. There are many methods for such estimation and to a large degree, the Rational Method has been calibrated accordingly.

We pursue t-c because Mulvaney's contribution depends upon it,. The Rational Method runoff model is such that Q from any rainfall intensity is a maximum when that rainfall intensity lasts as long or longer than t-c. Fig. 6 illustrates.

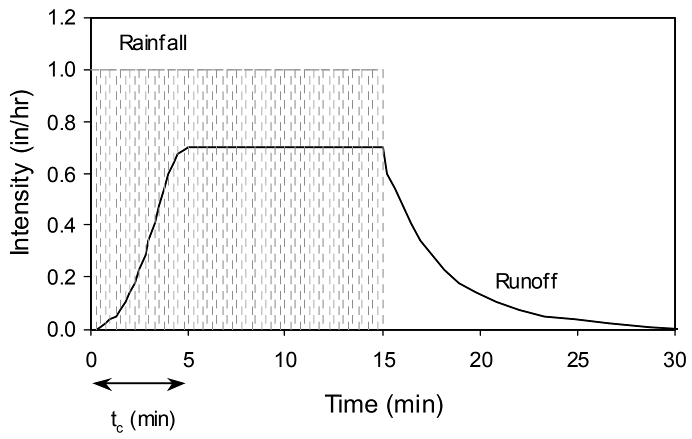

Figure 6. Rainfall/Runoff Intensities

FIG. 6. LONG t-c

Rain falls at 1.0 inch/hour for a 15-minute t-d. (Total rainfall is thus 0.25 inches, a value not used.) Time of concentration is 5 minutes. Five minutes after initiation of rainfall, the total watershed contributes to runoff at the outlet. Since Rational Method i-ave is constant, runoff intensity (inches/hour) or runoff rate Q (inches/hour times watershed area) is unchanged until the storm stops. C, by inspection, is 0.70. The hydrograph's falling limb then begins as the subareas nearest the outlet complete their drainage. If t-d were 6, 15 or 30 minutes, anything equalling or greater than the 5-minute t-c, peak rate would not alter.

Fig. 7 shows the hydrograph when t-d is only 4 minutes, less than t-c. The rising limb is the same as that of Fig. 6 for 4 minutes. When the storm stops, however, the falling limb begins. (Figs 5 and 6 are simplified in this aspect. To the degree that runoff is detained in the upper reaches, the hydrograph crest could be delayed.) Were the Rational Method applied to the entire basin (and it shouldn't be), Fig. 6's Q implies a 0.60 C.

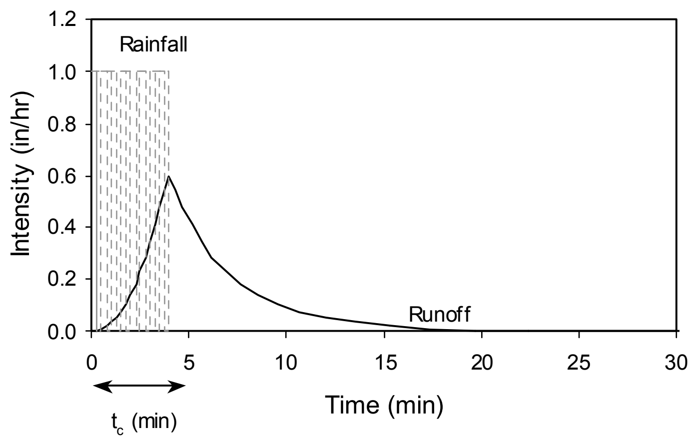

Figure 7. Rainfall/Runoff Intensities

FIG. 7. SHORT t-c

The Rational Method requires that t-d equals or exceeds t-c. Some observers take issue with the “or exceed” aspect of the rule. Such objection applies when the problem is viewed as one where i-ave isn’t given, but is itself a variable, per a PID function (Fig. 9, Eq. 9, Chapter 2). If that is the problem definition, the largest Q for a given T-r comes from the largest i-ave. The downward-sloped PID function has the greatest i-ave at the smallest allowable t-d, which is t-c and nothing more. To apply the Rational Method, find t-c (the remainder of this section) and use this value to maximize i-ave.

Contributing Area

Even Eq. 1’s A-cont term blends the boundary between art and science. Judgment is once more required. Debo and Reese (1995, p. 214) suggest several considerations regarding drainage area:

Existing and future flow paths along lot lines, at street intersections, diversions, etc.
Impact of regrading on flow direction and velocity.
Impact of drainage structures upon T-r.
Connection of impervious areas.
Suitability of C-comp.
Flow direction and rate for events in excess of the design event.

Observed Q’s generally don’t vary linearly with A-cont. Our example had A-cont’s of 100 and 500 acres and observed Q's of 25 and 75 cfs, not 25 and 125 cfs per Eq. 1. The Rational Method tends to overestimate Q when a C calibrated for a small watershed is applied to a larger area. Fig 8 illustrates the error. The error increases with latter’s size.

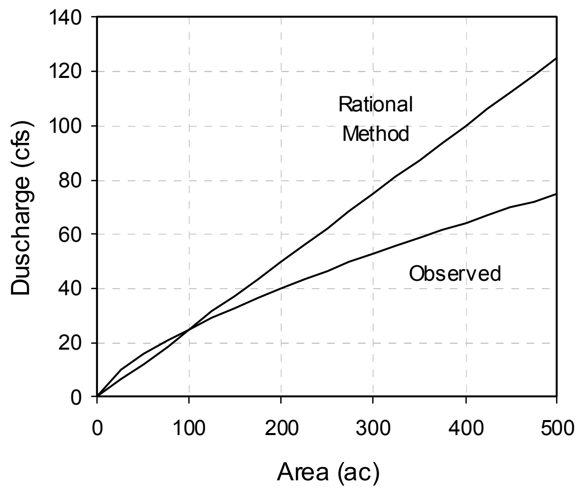

Figure 8. Discharge vs. Area

FIG. 8. DISCHARGE VS. AREA

While nothing in Eq. 1 mathematically imposes a maximum on A-cont, the Rational Method has an areal limit. Up to some limit (200 acres in the Fig. 8 example), linearizing a curvilinear process seems reasonable. Sources differ, as might be expected, as to this limit. Table 9 summarizes recommendations. Don’t presume that all recommendations are general in nature; some may be in particular context.

Table 9. Area Limits	
Area (acres)	Reference
20	Debo and Reese, 1995, p. 209
50 to 100	Wanielista, Kersten and Eaglin, 1997, p. 209
100 or much less	Debo and Reese, 1995, p. 194
100-200	Chow, 1964, p. 25-5
100-200	ASCE, 1992, p. 90
100-240	Singh, 1992, p 599
a few hundred	Bras 1990, p. 406
250, urban	ASCE, 1996, p. 580
300	ODOT, 1990, p. 9
325-625	Ponce, 1989, p.119
a few square km *	ASCE, 1996, p. 504
640	Gray, 1970, p. 8.2
640	USACE, 1996, p. 110
640	Veissman and Lewis, 1996, p. 318
2500, rural	Gupta, 1989, p. 621
*square km = 250 acres	

UDFCD (2001, p. 11) allows a 5-acre Rational Method minimum when Imp is less than 0.5 and 10 acres when Imp exceeds 0.5.

A key variable in the areal limit is the design storm. The more uniform the rainfall in both temporal and spatial sense, the greater the allowable A-cont. Keep Table 9 in mind as you review the assumptions reviewed at the end of this chapter. You may develop doubts about the larger values.

To this point, we've regularly appended the "-cont" to the A term to remind ourselves that Eq. 1 relates to contributing area, the land surface from which runoff is derived at any time. A minute after a storm begins, A-cont may be only a few square feet. At t-c, by definition, A-cont is the entire watershed.

AASHTO (1991, p. 7-22) warns, "in some cases, runoff from a portion of the drainage area which is highly impervious may result in a greater peak discharge than would occur if the entire area were considered. In these cases, adjustments can be made to the drainage area by disregarding those areas where flow time is too slow to add to the peak discharge."

ODOT (1990, p. 9) says, "On some combinations of drainage areas, it is possible that the maximum rate of runoff will occur from the higher intensity rainfall for periods less than the time of concentration for the total area, even though only a part of the drainage area may be contributing. This might occur where one part of the drainage area is highly impervious and has a short time of concentration, and another is pervious and has a much longer time of concentration."

These possibilities would violate the nature of the Rational Method if we could only look at total watershed area. By considering time-dependent A-cont, however, the Rational Method holds (or perhaps more honestly, has no more difficulties that before). Chow (1964 p. 20-9) designates what follows as the "Zone Principle". ASCE (1992, p. 94) calls what follows the "Contributing Area Method", but we'll reserve that title for a fuller hydrograph procedure in Section. 8.

<u>Isochrones</u>

Isochrones are contours joining watershed locations that share the same travel time (and thus t-c) to the basin outlet. Isochrones cannot cross. Isochrones cannot originate or terminate anywhere but on the watershed boundary (Singh, 1988, p. 130). Fig. 9 illustrates an example.

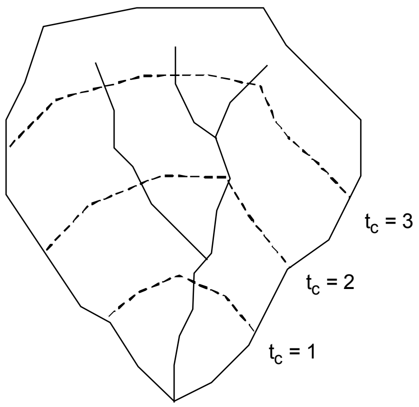

Figure 9. Isochrones

FIG. 9 ISOCHRONES

Isochrones are the basis for tracking A-cont and thus explaining the possibility of peak Q before watershed t-c. Table 10 is for a 20-acre watershed with isochrones determined for 5-minute time steps. At 5 minutes, the bottom 5 acres contribute runoff at the outlet. At 10 minutes, the next 6 acres of upstream watershed are contributing, etc. The third column shows cumulative A-cont. The fact that the entire watershed contributes runoff at 20 minutes makes t-c 20 minutes de facto. The fifth column shows the respective C-comp (Eq. 11) for the cumulative A-cont.

Table 10. Isochronal Analysis

t-d	A	A-cont	C	C-comp	i-ave	Q
(min)	(ac)	(ac)			(in/hr)	(cfs)
5	5	5	0.40	0.400	2.46	4.91
10	6	11	0.30	0.345	2.10	7.98
15	6	17	0.40	0.365	1.85	11.47
20	3	20	0.10	0.325	1.66	10.79
25		20		0.325	1.51	9.83
30		20		0.325	1.39	9.05
35		20		0.325	1.29	8.41
40		20		0.325	1.21	7.86
45		20		0.325	1.14	7.40
50		20		0.325	1.08	7.00
55		20		0.325	1.02	6.64
60		20		0.325	0.97	6.33

For an illustrative PDD, return to the illustrative 10-year event of Chapter 3. As we there noted, c is 20, e is 1.0, f is 15 and g is 0.7. The sixth column of Table 10 shows the thus-computed average intensities.

The last column is the Eq. 1 Q for the contributing area as duration increases. Fig 10 reveals the maximum Q. While the overall basin t-c is 20 minutes, because of the subareas and their respective C's, the 15-minute t-d produces the greatest Q. Change C of the upstream 3 acres from 0.10 to 0.30 and Q peaks at 20 minutes, not 15. Doing Table 10 by hand, you could stop at t-2, 20 minutes. We know from Section 4 know that after t-c, i-ave decreases, and thus Eq. 1 Q decreases.

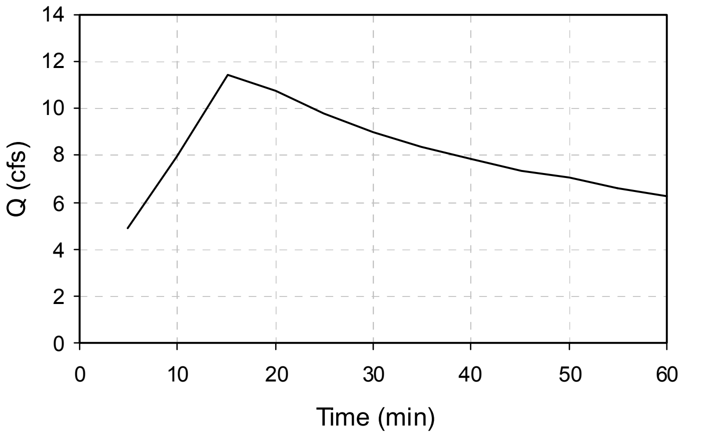

Figure 10. Isochronal Peak Discharge

FIG. 10. ISOCHRONAL QP

Fig. 10 is not a hydrograph, the topic of Chapter 12, though the axes correspond. Fig 10 shows Q's for i-ave over given durations, not i-inc . Fig 10 represents the Q appropriate to nested t-d's within the storm, not a clock. The 11.47 cfs peak runoff stems from the heaviest 15-minute rainfall, whenever it occurs.

When Q peaks before t-c, the discharge is said to peak at the "critical storm duration". Singh (1988, p. 141) pursues the theoretical basis. If you enjoy differential equations, we're solving Eq. 1 for increasing A-cont, changing C-comp and decreasing i-ave. Knowing A-cont as a function of time, C-comp as a function of A-cont and i-ave as a function of time, differentiate with respect to time and maximize Q.

The above example deals with sequential subareas. The 5-acre subarea is below the first 6-acre area, etc. What if outflows from two subareas converge at a mutual outlet, parallel subareas in schematic form? Were we doing a watershed simulation, perhaps with HEC-1 or HEC-HMS, we would simply add the two hydrographs. We can't just add the two Rational Method Q's, however, as we have no assurance that they occur at the same time.

Isochrones provide us a solution for parallel subareas. Analyze each subarea as we did above using isochrones of a constant time-step. These are computationally separate problems, though you need only compute the i-ave's once. Each subarea will have its own isochrones and the total may not be the same. Add the resulting Q columns of the two Table 10's to find the combined peak. This "split" watershed approach is used in more-complex hydrologic modeling as well. Connected impervious subareas are modeled for their hydrograph spike while the pervious watershed is still in transit to the outlet.

Wanielista, Kersten and Eaglin's (1997, p. 241) define the product of C and A-cont as "Equivalent Impervious Area". The equivalency conceptually adjusts watershed subareas as if all have a plastic surface. Such interpretation can save a few steps in some computations, but leads to errors when timing is considered (the t-c issue of Section 5). Leave this as an interesting thought, not an analytic shortcut.

In Chapter 12 we will use the Rational Method for hydrograph generation, extending the method from one of Fig XX peak flow estimation to one of runoff rate over time. Although the power of the Rational Method tends to wane at that level of hydrologic modeling, the method caries on, largely because many hydrologists never progress beyond Eq. 1. We will then compare Rational Method-based solutions to other hydrograph techniques. This current chapter, the computations to this point and the theoretical foundation to be discussed, will help you discern the relative merits in hydrograph modeling.

Assumptions, the Science of the Art

To this point, we have focused largely toward getting results: estimating C, calculating i-ave, determining A-cont. There have been many choices, in final rendition, your professional decisions. To the degree that you rely on experience, breadth of perspective and realistic judgment, you are practicing an art. You'll sense what probably fits together and what probably doesn't. If your exposure to the Rational Method is no more than that of this course, you at least are equipped to develop your talent. You already know more than many of your colleagues who've been cranking out Q's for years.

Our objective is not to practice the Rational Method by some strict scientific checklist. Under harsh light, it looks weak. Our objective is to anticipate the concerns that should be raised every time we put the method to use. By so anticipating, we can better formulate our solutions. We can sense were our results may be biased. We can look for other methods where the Rational Method is inappropriate.

Singh (1992, p. 598) lists eight Rational Method assumptions. Chow (1964, p. 14-7) finds the assumptions to hold best for paved areas with gutters and sewers of fixed hydraulic characteristics. As you consider each assumption, again think of how it relates to watersheds you know.

Assumption 1. "The peak rate of runoff is a maximum and is a direct function of the average rainfall intensity throughout the time of concentration. In other words, the computed discharge is the maximum that can occur for the selected rainfall intensity from that basin and that discharge occurs at the time of concentration and beyond."

Does i-ave adequately represent the real event? Debo and Reese (1995, p. 210) observe that convective storm cells in arid and semi-arid climates are relatively small with high, rapidly-varied intensity. The Rational Method may be inappropriate. When runoff seems to be more determined by simultaneous intensity than by average rate, Assumption 1 is in question.

Assumption 2. "The maximum discharge resulting from a rainfall intensity equal to or greater than the time of concentration is a simple, direct function of the rainfall. In other words, runoff [rate] is directly proportional to rainfall."

Is C sufficiently constant? Assumption 2's flag is the "directly proportional" assertion. As we've seen, C is not independent of the storm. Virtually no other hydrologic model presumes such proportionality. The Rational Method works only to the extent we have an empirical C appropriate to the situation.

Assumption 3. "The frequency of the peak discharge is the same as the frequency of rainfall. This assumption is not strictly correct, but does not create significant problems for the limited purposes of this method."

Does the 100-year discharge come from the 100-year storm? Veissman and Lewis (1996, p. 317) find Assumption 3 reasonable in the design range. FHWA (1984, p. 137) indicates that approximately 25 percent of "modest to important" federal projects are sized by assuming identical rainfall and runoff frequencies. Peak discharge has roughly the same probability of occurring as the corresponding storm if the storm isn't unusually large. Lognormal probability plots of precipitation and runoff diverge for extreme events. The Rational Method works better for 2-year events than 100-year events.

Assumption 4. "The relation between peak discharge and the drainage area is the same as the relations between peak discharge, intensity, and duration of rainfall. This means that the drainage basin is considered linear."

Should C, i-ave and A-cont be simply multiplied? "Linear" is used in the sense of mathematical function. Given a large Q, i-ave and A-cont data set and taking log transforms, Assumption 4 suggests that multiple linear regression without additional coefficients should fit the C-comp to

Eq. 1. Although most C's stem from less-objective quantification, years of experience have evolved approximate fits. While Assumption 4 may not be scientific, we can work with the art.

We'll not trust even approximate linearity, however, for analyses that fall beyond the range for which the Rational Method was developed. Recall Fig. 19 where large A-cont's produced increasingly large error.

Assumption 5. "The coefficient of runoff, C, is the same for storms of various frequencies. This means that all the losses on the drainage basin are a constant. This assumption may be reasonable for a drainage basin covered with an impervious surface, but not for other drainage surfaces."

Is C independent of T-r? We know it's not. As a watershed is made wetter by a larger storm, rainfall of the same intensity will find less capacity for abstraction. The fortunate side of this erroneous Assumption 5 is that corrections are available. A dilemma is that we have T-r corrections for both C and i-ave. Applying both may be excessive.

Assumption 6. "The coefficient of runoff is the same for all storms on the drainage basin regardless of antecedent moisture condition. Variation in the runoff coefficient is often a function of drainage area. This implies that the drainage basin for which the assumption is valid is necessarily small."

Does it matter if it rained yesterday? Assumption 6 is woefully in error and the deficiency lacks a clean remedy. The safe approach is to limit the Rational Method to typical soil moisture conditions.

Assumption 7. "Rainfall remains constant over the entire watershed during the time of concentration. The significance of this assumption is that because of the spatial variability of rainfall, the drainage area for which the rational method will apply is limited."

How spread out is the storm? Spatially distributed rainfall is less of an issue for small watersheds than large ones and less of an issue for frontal precipitation than for convective or orographic events. Isohyetal maps resolve Assumption 7, positively or negatively. While you may lack such a map, common observation usually indicates how broadly rain spreads over a region.

Assumption 8. "Runoff occurs nearly uniformly from all parts of the watershed. This means that the runoff coefficient must be nearly the same over the entire drainage basin. This assumption is less likely to be valid as the drainage-basin size increases."

How uniform is the watershed? To resolve the adequacy of Assumption 8, walk the watershed, preferably when it's raining. The issue isn't that everything looks the same, but that whatever is distributed, say lawns and pavements, is distributed with reasonable consistency over the entire area. If you aren't willing to walk, don't use the Rational Method.